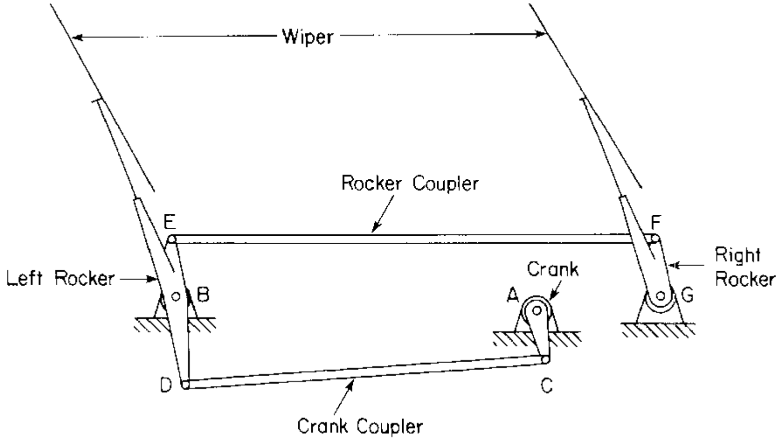
ME601
Kinematics and Dynamics of Machine Systems

Machines and Mechanisms

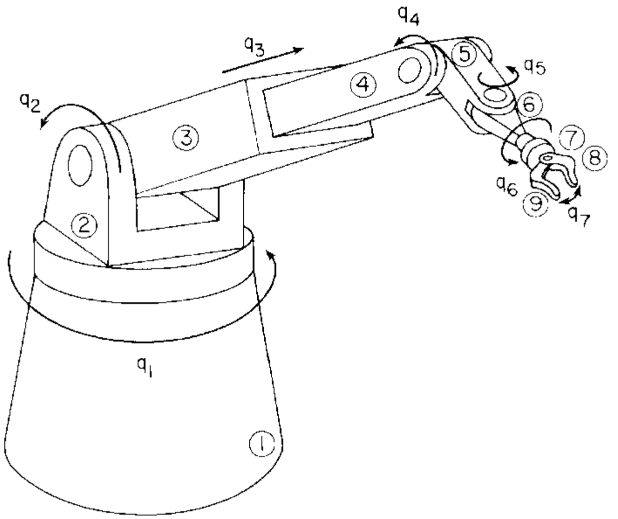
- A useful working definition of a **mechanism** is *A system of elements arranged to transmit* **motion** *in a predetermined fashion.*
- On the other hand, a ***machine*** is *A system of elements arranged to transmit* **motion and energy** *in a predetermined fashion.*

Inverse Dynamics Analysis

- It is a hybrid between Kinematics and Dynamics
- Basically, one wants to find the set of forces that lead to a certain desirable motion of the mechanism
- Your bread and butter in Controls…



Windshield wiper mechanism



Robotic Manipulator

Geometric Vectors

- What is a Geometric Vector?
  - A quantity that has two attributes:
    - A direction
    - A magnitude

- VERY IMPORTANT:
  - Geometric vectors are quantities that exist independently of any reference frame

- ME451 deals almost entirely with planar kinematics and dynamics
  - We assume that all the vectors are defined in the 2D plane

Geometric Vectors: Operations

- What can you do with geometric vectors?
  - Scale them
  - Add them (according to the parallelogram rule)
    - Addition is commutative
  - Multiply two of them
    - Inner product (leads to a number)
    - Outer product (leads to a vector, perpendicular on the plane)
  - Measure the angle θ between two of them

Unit Coordinate Vectors (short excursion)

- Unit Coordinate Vectors: a set of unit vectors used to express all other vectors

- In this class, to simplify our life, we use a set of two orthogonal unit vectors

- A vector **a** can then be resolved into components a_x and a_y , along the axes *x* and *y*

$$\mathbf{a} = a_x\vec{\mathbf{i}} + a_y\vec{\mathbf{j}}$$

- Nomenclature: a_x and a_y are called the Cartesian components of the vector

- Notation convention: throughout this class, vectors/matrices are in bold font, scalars are not (most often they are in italics)

Geometric Vectors: Operations

- Dot product of two vectors

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a\, b \cos\theta(\vec{\mathbf{a}}, \vec{\mathbf{b}})$$

- Regarding the angle between two vectors, note that

$$\theta(\vec{\mathbf{a}}, \vec{\mathbf{b}}) \neq \theta(\vec{\mathbf{b}}, \vec{\mathbf{a}}) \qquad \theta(\vec{\mathbf{a}}, \vec{\mathbf{b}}) = 2\pi - \theta(\vec{\mathbf{b}}, \vec{\mathbf{a}})$$

- The dot-product of two vectors is commutative

- Since the angle between coordinate unit vectors is $\pi/2$:

$$\vec{\mathbf{i}} \cdot \vec{\mathbf{i}} = \vec{\mathbf{j}} \cdot \vec{\mathbf{j}} = 1 \qquad \vec{\mathbf{i}} \cdot \vec{\mathbf{j}} = \vec{\mathbf{j}} \cdot \vec{\mathbf{i}} = 0$$

Derivatives and the shapes of graphs

<u>Increasing / Decreasing Test:</u>

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

<u>Example</u>: Find where the function $f(x) = x^3 - 1.5x^2 - 6x + 5$ is increasing and where it is decreasing.

Solution: $f'(x) = 3x^2 - 3x - 6 = 3(x + 1)(x - 2)$

$f'(x) > 0$ for $x < -1$ and $x > 2$;

thus the function is increasing on $(-\infty, -1)$ and $(2, \infty)$.

$f'(x) < 0$ for $-1 < x < 2$;

thus the function is decreasing on $(-1, 2)$.

The First Derivative Test: Suppose that c is a critical number of a continuous function f.

(a) If f' is changing from positive to negative at c, then f has a local maximum at c.

(b) If f' is changing from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c, then f has no local maximum or minimum at c.

Example(cont.): Find the local minimum and maximum values of the function $f(x) = x^3 - 1.5x^2 - 6x + 5$.

Solution: $f'(x) = 3x^2 - 3x - 6 = 3(x + 1)(x - 2)$

f' is changing from positive to negative at -1 ; so $f(-1) = 8.5$ is a local maximum value ;

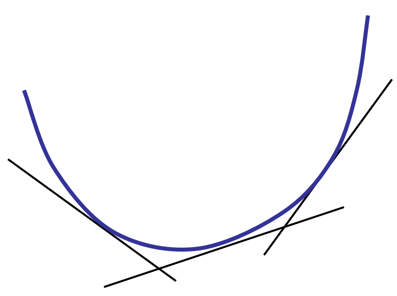
f' is changing from negative to positive at 2 ; so $f(2) = -5$ is a local minimum value.

Concave upward and downward

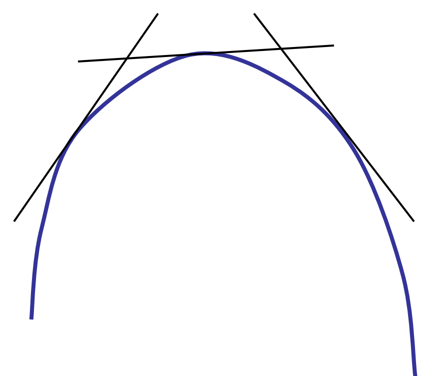
Definition:

(a) If the graph of f lies above all of its tangents on an interval, then f is called **concave upward** on that interval.

(b) If the graph of f lies below all of its tangents on an interval, then f is called **concave downward** on that interval.



Concave upward



Concave downward

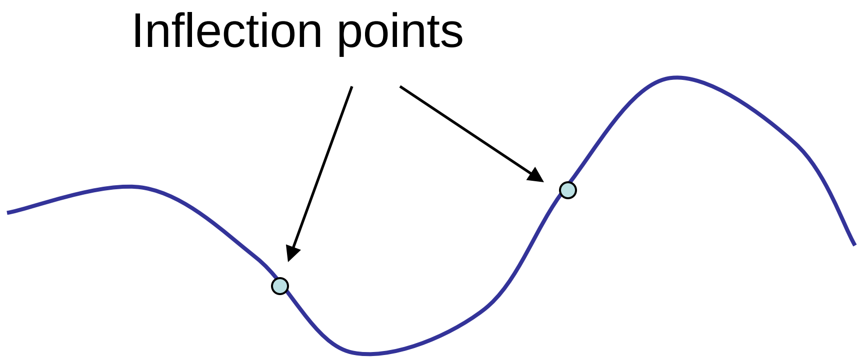
Inflection Points

Definition:

A point P on a curve $y = f(x)$ is called an **inflection point** if

f is continuous there and the curve changes

- from concave upward to concave downward *or*
- from concave downward to concave upward at P.



What does f'' say about f?

Concavity test:

(a) If $f''(x) > 0$ for all x of an interval, then the graph of f is concave upward on the interval.

(b) If $f''(x) < 0$ for all x of an interval, then the graph of f is concave downward on the interval.

Example(cont.): Find the intervals of concavity of the function $f(x) = x^3 - 1.5x^2 - 6x + 5$.

Solution: $f'(x) = 3x^2 - 3x - 6$ $\quad f''(x) = 6x - 3$

$f''(x) > 0$ for $x > 0.5$, thus it is concave upward on $(0.5, \infty)$.

$f''(x) < 0$ for $x < 0.5$, thus it is concave downward on $(-\infty, 0.5)$.

Thus, the graph has an inflection point at $x = 0.5$.

Using f'' to find local extrema

<u>The second derivative test</u>: Suppose f is continuous near *c*.

(a) If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum at *c*.

(b) If $f'(c) = 0$ and $f''(c) < 0$ then f has a local maximum at *c*.

<u>Example(cont.)</u>: Find the local extrema of the function $f(x) = x^3 – 1.5x^2 – 6x + 5$.

Solution: $f'(x) = 3x^2 – 3x – 6 = 3(x + 1)(x - 2)$,

so $f'(x) = 0$ at $x=-1$ and $x=2$

$f''(x) = 6x - 3$

$f''(-1) = 6*(-1) – 3 = -9 < 0$, so $x = -1$ is a local maximum

$f''(2) = 6*2 – 3 = 9 > 0$, so $x = 2$ is a local minimum

Summary of what y' and y'' say about the curve

First derivative:

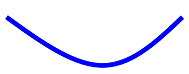
y' is positive → Curve is rising.

y' is negative → Curve is falling.

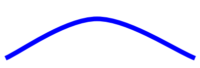
y' is zero → Possible local maximum or minimum.

Second derivative:

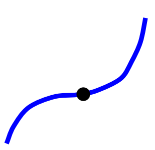
y'' is positive → Curve is concave up.



y'' is negative → Curve is concave down.



y'' is zero → Possible inflection point (where concavity changes).



Example(cont.): Sketch the curve of $f(x) = x^3 - 1.5x^2 - 6x + 5$.

From previous slides,

$f'(x) > 0$ for $x < -1$ and $x > 2$; thus the curve is increasing on $(-\infty, -1)$ and $(2, \infty)$.

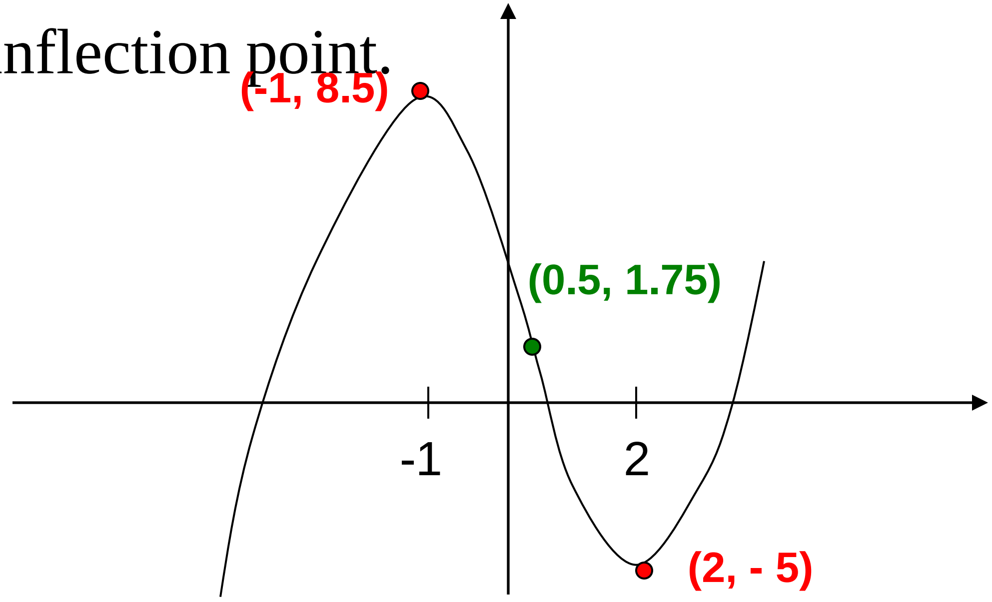
$f'(x) < 0$ for $-1 < x < 2$; thus the curve is decreasing on $(-1, 2)$.

$f''(x) > 0$ for $x > 0.5$; thus the curve is concave upward on $(0.5, \infty)$.

$f''(x) < 0$ for $x < 0.5$; thus the curve is concave downward on $(-\infty, 0.5)$

(-1, 8.5) is a local maximum; (2, -5) is a local minimum.

(0.5, 1.75) is an inflection point.



Curve Sketching

<u>Guidelines for sketching a curve</u>:

A. Domain

Determine D, the set of values of x for which $f(x)$ is defined

B. Intercepts

- The y-intercept is $f(0)$
- To find the x-intercept, set y=0 and solve for x

C. Symmetry

- If $f(-x) = f(x)$ for all x in D, then f is an even function and the curve is symmetric about the y-axis
- If $f(-x) = -f(x)$ for all x in D, then f is an odd function and the curve is symmetric about the origin

D. Asymptotes

- Horizontal asymptotes
- Vertical asymptotes

Guidelines for sketching a curve (cont.):

E. Intervals of Increase or Decrease

- f is increasing where $f'(x) > 0$
- f is decreasing where $f'(x) < 0$

F. Local Maximum and Minimum Values

- Find the critical numbers of f ($f'(c)=0$ or $f'(c)$ doesn't exist)
- If f' is changing from positive to negative at a critical number c, then $f(c)$ is a local maximum
- If f' is changing from negative to positive at a critical number c, then $f(c)$ is a local minimum

G. Concavity and Inflection Points

- f is concave upward where $f''(x) > 0$
- f is concave downward where $f''(x) < 0$
- Inflection points occur where the direction of concavity changes

H. Sketch the Curve